Black Rose Technology

CRATER ANALYSIS OF 2022-03-13 ERBIL MISSILE STRIKES

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A NUMERICAL REVIEW OF THE CRATER IMPACTS VISIBLE IN SATELLITE IMAGERY FROM THE 2022-03-13 ATTACK ON SHEIKH BAZ' HOUSE IN THE OUTSKIRTS OF ERBIL.

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CRATER ANALYSIS OF 2022-03-13 ERBIL MISSILE STRIKES

There have been, as of this writing, two attacks on the Erbil region reportedly carried out by the IRGC regional targets. Both are believed to have been carried out with Fateh-family missiles. An analysis adds some useful real-world data to understanding key parameters of these rapidly proliferating devices.

Only the 2022-03-13 attack, for now, is analyzed in detail because there exist clear satellite images of impact craters.

This analysis is relevant to the 2024-01-15 attack on Peshraw Dizayee, just 5 km further up the Erbil-Pirmam road (36.305, 44.132222), on which the US consulate is also located, and which was reportedly carried out with the same family of missiles. The بیسیمچی مدیا telegram channel reported "Announcement No. 4 of the Islamic Revolutionary Guard Corps: *4 missiles from the west and 7 more missiles from the northwest of the country to the Mossad spy headquarters in the Kurdistan region of Iraq.*" Eight explosions were generally reported (counts vary) and one missile can be seen in video footage clearly disintegrating before impact, so 9 of 11 are accounted for on this target, consistent with expected reliability.

1 THE FATEH FAMILY

There are a few generations of the Fateh enumerated in public data of the Fateh family of missiles and close relatives, all of which are single-stage solid-propellant transporter-erector-launcher (TEL) based SRBMs (and one MRBM). The Fateh-110 family (1997) was developed from the Zelzal-2 (1988), which in turn was derived from the Soviet Luna-M/FROG-7, and may have derived some additional details from the Chinese DF-11 for the Fateh-110B variant forward. This analysis omits the anti-ship variants of the same family as unlikely to be relevant.

Reports indicate that both the 2022-03-13 (Baz) and the 2024-01-15 (Dizayee) attacks were carried out with Fateh-110 missiles, but the specific iteration has not been reported. From this analysis it is likely the Baz attack was carried out with either 110B or, more consistent with the data, Zolfaghar variants.

Without access to clear satellite imagery or ground access to measure crater impacts, it is premature to review the Dizayee attack. The data available indicates that 8 of 11 missiles launched successfully hit their targets for a 27% failure rate, which is quite good. Superficially, the one aerial shot found shows all but one impact within the target footprint, suggesting a small CEP, however this is extremely premature as impacts may well be out of frame and the image may have been edited.

TABLE 1: FATEH 110 VARIANTS

Variant	Date	km	kg	CEP m	Mach
Fateh 110 G1	2002	200	650	600	4
Fateh 110A G2	2004	250	450	600	4
Fateh 110B G3	2010	300	650	250	3
Fateh 110D1 G4	2012	300	650	10	3
M-600 (Mod-A-110B)	2010	250	450	50	4
Fateh 313 G5	2015	500	380	2	5
Zolfaghar G6	2017	750	579	100	8
Dezful G7	2019	1,000	700	5	7
Fath 360 G7 mini	2020	100	150	30	4

2 2022-03-13 ATTACK: ERBIL 01:22

The 2022-03 google imagery of the remains of the villa of Baz Karim Barzanji, CEO of KAR Group, clearly shows impact craters, apparently 6, from the attack. Reports on the number of missiles launched vary from 10-12 while there is evidence of at least 6 impacts, which equates to a failure rate of 40-50%, higher than the Dizayee attack's missiles, if this data holds up, we may conclude that the later attack was carried out with a more advanced generation of missile than the earlier.

This is consistent with expectations, no missile system has 100% success, not even Western systems. Russian missiles in Ukraine have been reported as having a failure rate as high as 60%, an Iraqi program of similar missile types had a reported failure rate of 30%. It is within the normal range for only 40-70% of fired missiles perform on target given the range of failures from launch to detonation that might interfere with success.

From the 6 visible impact craters, a visual analysis using Google Earth (KML available) yields the following data. The HMX kg column is the estimated yield from the crater apparent radius, derived as detailed below.

TABLE 2: 2022-03-13 CRATER DATA

Impact	Δ m	Δ Xm	Δ Ym	Crater m	HMX kg
1	188	157	103	5.6	394
2	305	236	193	5.3	334
3	95	95	-8	4.8	247
4	56	-52	-21	4.9	263
5	18	-7	17	NA	NA
6	13	-13	3	NA	NA

3 CEP ANALYSIS

Circular Error Probability (CEP) is the circular diameter within which 50% of projectiles are expected to land. Using CEP = 0.5887(STDEV(X) + STDEV(Y)), the estimated CEP of this attack is **116 m**, which most closely matches the Zolfaghar missile and is plausible for the Fateh 110-B, it is either significantly better or worse than the other rockets in the family.

4 YIELD ANALYSIS

Yield estimates from this analysis range from 263-394 kg with a mean of **310 kg**. All Fateh variants can deliver at least a 300 kg payload except the latest, the Fath 360, which suggests the warheads were under-filled. One reason for under-fill might be related to an analysis by Geoff Forden at Arms Control Wonk which found that Iranian warheads would be unstable in flight at full-fill. Rockets may also reasonably be under-filled to increase range or to conserve HE. The estimated US price of HMX is $100/kg, a meaningful cost component at an estimated per missile price of $100-300k. At $10k/100kg; an underfill of 269 kg of HMX between the estimated yield of 310kg and the reported payload capacity of the Zolfighar's 579 kg yields a savings of $26,900 per missile or $269,000 over the entire launched salvo.

5 ESTIMATING YIELD FROM CRATER SIZE

Yield estimates based on crater size are well established, including Bjelovuk, 2015, Estimation of the Explosive Mass Based on the Surface Explosion Crater on Asphalt, and Cooper, Jr, H F. 1976, Estimates of crater dimensions for near-surface explosions of nuclear and high-explosive sources, and Chowdhury, 2015 Characterizing Explosive Effects on Underground Structures. The standard reference remains Cooper, P. W., Explosives Engineering, 1937 and the formulas from that reference were used in the calculations presented in this document.

However, the relevant equations are written in a mix of English and Metric units and solve for expected crater size from a known explosive mass; rewriting them to be fully metric and to solve for warhead size as below yielded the above estimated warhead sizes and may be useful to others for strike analysis.

Assuming Iran is using HMX/Octagon as their HE material, it is reported as having a density of 1.91 g/cm3 and a detonation velocity of 9100 m/s. We also assume that the soil is typical alluvial soil with a E_{CR} of 0.5. Equation 8, derived below, yields our estimate of the warhead HMX masses from the measured crater apparent radii.

5.1 ENERGY PROPORTIONALITY

From Explosives Engineering § 29.2 (Cooper, 1937), we learn the base equation estimates the crater radius, R_a, is proportional to a soil constant, K, and the cube root of the explosive energy, $E^{0.33}$ or:

$$R_a = K\sqrt[3]{E} \tag{1}$$

Madal et. al in their 2021 paper Surface and Buried Explosions: An Explorative Review with Recent Advances found that a slight modification to the root fit data better in most cases.

$$R_a = K\sqrt[3.4]{E} \tag{2}$$

However, in our analysis we find this exponent results in a significant overestimate of yield and so continue to use the $\sqrt[3]{}$ value that dominates the literature.

5.2 SOLVING FOR MASS

Cooper expands K and E by fitting to empirical data from testing with a variety of explosives and soil samples and derives a fairly general equation (using the original Cooper exponent). The solution requires use of an empirically derived constant, E_{CR} in ft³/ton (but effectively unitless) for Cratering Efficiency, which as to be estimated by soil appearance in this study.

$$R_a = (0.46 + 0.027 * P_{CJ})\sqrt[3]{(2E_{CR}W)} \tag{3}$$

PCJ is the Chapman-Jouguet Pressure in GPa (whether solved in English or Metric units) and is computed as:

$$P_{CJ} = \rho D^2\left(1 - 0.7125 * \rho^{0.04}\right) \tag{4}$$

Where D is the detonation velocity in km/sec (English or Metric) and ρ is the density of the explosive in g/cm³, also whether English or Metric.

W is normalized to TNT in pounds and has to be corrected for other explosives based on the detonation velocity:

$$W = W_{HE}\left(\frac{D^2}{48.3}\right) \tag{5}$$

Expanding the equation fully to solve for the apparent crater diameter in feet for a high explosive mass in pounds yields:

$$R_a = \left(0.46 + 0.027\left(\rho D^2\left(1 - 0.7125\rho^{0.04}\right)\right)\right) \times \sqrt[3]{2E_{CR}W_{HE}\left(\frac{D^2}{48.3}\right)} \tag{6}$$

Which can be corrected to metric units of explosive in kilograms and apparent crater radius in meters by simply converting K from ft/lb⅓ to m/kg⅓ by multiplying by 0.397:

$$R_a = 0.397 \times \left(0.46 + 0.027\left(\rho D^2\left(1 - 0.7125\rho^{0.04}\right)\right)\right) \times \sqrt[3]{2E_{CR}W_{HE}\left(\frac{D^2}{48.3}\right)} \tag{7}$$

A single line ASCII representation: R=0.397*(0.46+0.027(rD^2(1-0.7125r^0.04)))*(2E*W*(D^2/48.3))^(1/3)

Solving for W_{HE} in metric units of meters and kilograms is written as:

$$W_{HE}=\frac{\left(\frac{R_a}{0.397\times(0.46+0.27(\rho D^2(1-0.7125\rho^{0.04})))}\right)^3}{2E_{CR}\times\left(\frac{D^2}{48.3}\right)} \tag{8}$$

5.3 VARIABLES AND UNITS

TABLE 3: VARIABLES

Symbol	Description	Units	Value
R_a	Apparent crater radius	feet or meters	measured
ρ	Explosive density	g/cm^3	1.91 (HMX)
D	Detonation velocity	km/s	9.1 (HMX)
E_{CR}	Cratering efficiency	ft^3/ton	0.5 (see table)
P_{CJ}	Chapman-Jouguet Pressure	Gpa	42.52 (computed)
W	Mass of high explosive, TNT	lbs or kg	intermediate
W_{HE}	Mass of high explosive	lbs or kg	solution
K	Constant for ground and explosive type	ft/lb^3	1.61 (computed)

The ground constant is determined empirically and is estimated based on observation. From Cooper (1937), table 29.1, values for E_{CR} for various soil types are provided in an appendix.

Additional data may be found in Blasting and blast effects in cold regions, Part III, Mellor (1989).

The computations for yield and CEP in spreadsheet form are available here, a KMZ file of the site analyzed is available here.

6 APPENDIX: EQUATIONS

LaTeX code for the equations used:

EQ 1: {{R}_{a}}{=}K{\sqrt[{3}] {E}}

EQ 2: {{R}_{a}}{=}K{\sqrt[{3.4}] {E}}

EQ 3: {{{R}_{a}}=}\left ({0.46+0.027*{{P}_{CJ}}} \right){\sqrt[{3}] {\left ({2{{E}_{CR}}W} \right)}}

EQ 4: {{{P}_{CJ}}=}ρ{{D}^{2}}\left ({1-0.7125*{{ρ}^{0.04}}} \right)

EQ 5: {W=}{{W}_{HE}}\left ({{\frac {{D}^{2}} {48.3}}} \right)

EQ 6: {{R}_{a}}{=}\left ({0.46+0.027\left ({ρ{{D}^{2}}\left ({{1-0.7125{ρ}^{0.04}}} \right)} \right)} \right){{\times }\sqrt[{3}] {2{{E}_{CR}}{{W}_{HE}}\left ({{\frac {{D}^{2}} {48.3}}} \right)}}

EQ 7: {{R}_{a}}{=0.397{\times }}\left ({0.46+0.027\left ({ρ{{D}^{2}}\left ({{1-0.7125{ρ}^{0.04}}} \right)} \right)} \right){{\times }\sqrt[{3}] {2{{E}_{CR}}{{W}_{HE}}\left ({{\frac {{D}^{2}} {48.3}}} \right)}}

EQ 8: {{W}_{HE}}{=}{\frac {{\left ({{\frac {{R}_{a}} {0.397{\times \left ({0.46{+0.27\left ({ρ{{D}^{2}\left ({1-0.7125{{ρ}^{0.04}}} \right)}} \right)}} \right)}}}} \right)}^{3}} {2{{E}_{CR}}{\times }\left ({{\frac {{D}^{2}} {48.3}}} \right)}}}

7 APPENDIX: CRATERING EFFICIENCY

TABLE 4: SOIL TYPE CRATERING EFFICIENCY

Soil composition	**Cratering Efficiency E_{CR} (ft^3/ton)**
Clay soil/shale (water saturated)	2.00
Clay soil/shale, claystone	0.95
Galacial Soil	0.75
Clay soil/shale	0.55
Alluvial soil	0.50
Sandy clay soil	0.475
Playa	0.45
Sandstone	0.25
Basalt-Granite	0.20